General Relativity For Tellytubbys

Miscellaneous Mathematics

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This section is a refresher to bits of stuff that, if you don't already know, you had better resign yourself to flipping hamburgers for one or two years yet, before you have any hope of inventing a new Warp Drive. Not all of the details will be filled in cos I can not be bothered.

Derivative Chain Rule

Consider a function that is a function of a function i.e.

$$F = F(V(x))$$

Forming the derivative of this gives

$$\frac{dF}{dx} = \frac{F(V(x+\Delta x) - F(V(x))}{\Delta x}$$

$$as \lim \Delta x -> 0$$

$$\frac{dF}{dx} = \frac{F(V(x+\Delta x) - F(V(x))}{\Delta v} \cdot \frac{\Delta v}{\Delta x}$$

$$\lim \Delta x -> 0$$

$$\frac{dF}{dx} = \frac{\partial F}{\partial V}\frac{\partial V}{\partial x}$$

Now suppose that the function is a function of more then one function

$$F = F(V(x), U(x))$$

With a bit of piddling about, using the standard derivative of product rules, the partial derivative can be found to be

$$\frac{dF}{dx} = \frac{\partial F}{\partial V}\frac{\partial V}{\partial x} + \frac{\partial F}{\partial U}\frac{\partial U}{\partial x}$$

Or in more general terms, for an arbitrary number of functions, F can be written as:

$$F = F(X^{\alpha}(x))$$

then

$$\frac{d\,\mathrm{F}}{dx} = \frac{\partial\,\mathrm{F}}{\partial \mathrm{X}^1}\frac{\partial \mathrm{X}^1}{\partial x} + \frac{\partial\,\mathrm{F}}{\partial \mathrm{X}^2}\frac{\partial \mathrm{X}^2}{\partial x}\frac{\partial\,\mathrm{F}}{\partial \mathrm{X}^3}\frac{\partial \mathrm{X}^3}{\partial x} + \ldots$$

This can be written more compactly as:

$$\frac{d\,\mathrm{F}}{dx} = \frac{\partial\,\mathrm{F}}{\partial \mathrm{X}^\alpha}\frac{\partial \mathrm{X}^\alpha}{\partial x}$$

Where it is now assumed that repeated index's in a product will be summed, even though no sigma sign is shown.

Finally, if F is a set of function of set of variables x, y, z, one can generally write

$$\mathrm{F} = \mathrm{F}(\mathrm{X}^\alpha(x^1, x^2, x^3, \ldots))$$

or

$$\mathrm{F} = \mathrm{F}(\mathrm{X}^\alpha(x^\beta))$$

Then, it can be written

$$\frac{d\,\mathrm{F}}{dx^\beta} = \frac{\partial\,\mathrm{F}}{\partial \mathrm{X}^\alpha}\frac{\partial \mathrm{X}^\alpha}{\partial x^\beta}$$

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